MSC - Math - 5yr First Sem 2015

Roll No. .....

(12/15-I)

Math (5 year) 2015

3151

M. Sc. (5 Years Course) EXAMINATION

(For Batch 2010 & Onwards)

(First Semester)

MATHEMATICS

MHT-1101

Algebra

Time : Three Hours *Maximum Marks :* 40

Note : **Section A** (Compulsory Section) Attempt *all* the questions.

Section B : Attempt any *five* questions out of eight.

Section C : Attempt any *two* questions out of four.

Section A

1. Apply Descarte's rule of signs to discuss the nature of the roots of the equation : 2

$$3x^4 + 12x^2 + 5x - 4 = 0.$$

2. Prove that the eigen values of a diagonal matrix are the diagonal elements of the matrix. 2

3. If A is Hermition, then show that iA is Skew-Hermitian. 2

4. Remove the second term from the equation : 2

$$x^4 + 4x^3 + 2x^2 - 4x - 2 = 0$$

Section B

Note : Attempt any *five* questions in about **100** words each.

5. Prove that the characteristics roots of a unitary matrix are of unit modulus. 4

6. If α, β, γ are roots of the equation $(a-x)(b-x)(c-x)+1=0$, then prove that a, b, c are the roots of the equation $(x-\alpha)(x-\beta)(x-\gamma)+1=0$. 4

7. Solve the equation $x^3-27x+54=0$ by Cardon's Method. 4

8. Express :

$$A=\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$$

as the sum of symmetric and a skew-symmetric matrix. 4

9. A square matrix A is singular if and only if its columns are linearly dependent. 4

10. If α, β, γ are roots of $x^3-x-1=0$. Show by transformation technique that : 4

$$\frac{1+\alpha}{1-\alpha}+\frac{1+\beta}{1-\beta}+\frac{1+\gamma}{1-\gamma}=-7.$$

11. Find the minimal polynomial of the matrix

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

and show that it is derogatory.

12. Determine definiteness of the following quadratic form in R^3 with the help of leading principal minors : 4

$$6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1 x_2 - 2x_2 x_3 + 4x_3 x_1$$

Section C

13. Apply Descarte's Method to solve the equation : 6

$$x^4 - 4x^3 + 5x + 2 = 0$$

14. Solve the equation

$$x^5 - x^4 - 2x^3 + 2x^2 + x - 1 = 0$$

which has multiple roots. 6

15. For what value of λ and μ, the system of equation : 6

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

has :

(a) No solution

(b) A unique solution

(c) An infinite number of solution.

16. Prove that the rank of the product of two matrices A and B cannot exceed the rank of either matrix. 6